

Coupled Electromechanical Response of Composite Beams With Embedded Piezoelectric Sensors and Actuators

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June 1994

Prepared for
Lewis Research Center
Under Cooperative Agreement NCC3-208



National Aeronautics and
Space Administration

N94-33899

Unclass

G3/24 0012103

(NASA-CR-195313) COUPLED
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Final Report (Ohio Aerospace
Inst.) 20 p

COUPLED ELECTROMECHANICAL RESPONSE OF COMPOSITE BEAMS WITH EMBEDDED PIEZOELECTRIC SENSORS AND ACTUATORS

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SUMMARY

Unified mechanics are developed with the capability to model both sensory and active composite laminates with embedded piezoelectric layers. A discrete-layer formulation enables analysis of both global and local electromechanical response. The mechanics include the contributions from elastic, piezoelectric, and dielectric components. The incorporation of electric potential into the state variables permits representation of general electromechanical boundary conditions. Approximate finite element solutions for the static and free-vibration analysis of beams are presented. Applications on composite beams demonstrate the capability to represent either sensory or active structures, and to model the complicated stress-strain fields, the interactions between passive/active layers, interfacial phenomena between sensors and composite plies, and critical damage modes in the material. The capability to predict the dynamic characteristics under various electrical boundary conditions is also demonstrated.

INTRODUCTION

In the verge of continuous competing requirements for improving the weight, interdisciplinary performance, temperature stability, versatility, and reliability of propulsion and aerospace components, the development of a new generation of composite materials, often called "intelligent/smart composite materials" is receiving growing attention. Yet, the tremendous potential of intelligent materials and structures remains in many aspects unexplored, and new issues unique to these materials remain to be addressed. Among them, the opportunities to develop smart composites with the inherent capability to adapt their static and dynamic response, or sense the type, location, and extent of eminent damage (health monitoring) need to be explored, as they will further improve the performance and reliability of the composite under the adverse loading conditions imposed in current and future mission requirements (Venneri and Noor, 1992). However, these objectives require the development of admissible mechanics entailing capabilities to represent the electromechanical state of the structure, the complicated stress-strain fields in sensory/active composites, coupled electromechanical behavior of the sensors/actuators (represented in this paper by embedded piezoelectric layers), interfacial phenomena between the sensors and passive composite plies, and critical damage modes in the laminate. Consequently, the present paper presents recent analytical developments which address these issues.

Piezoelectrics have been employed for years in a variety of transducers, but their use as distributed sensors/actuators was limited until the mid-1980's. Bailey and Hubbard (1985) and Fanson and Chen (1986) demonstrated

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the possibility of using piezoelectric materials for beam vibration control. Crawley and de Luis (1987) developed piezoelectric elements for placement either on the surface or embedded within structural laminated beams. Single-layer piezoelectric laminate theories based on Kirchhoff-Love and the induced strain concept have been developed and in many cases experimentally correlated and applied to vibration (Lee, 1989, 1990, 1991; Lazarus and Crawley, 1989; and Dimitriadis et al., 1989) and noise control problems (Wang et al., 1989). Additionally, Wang and Rogers (1991) using assumptions of lamination theory presented a model for spatially distributed, small sized induced strain actuators embedded at any location of the laminate. Tzou and co-workers (Tzou, 1989 and Tzou and Gadre, 1989) developed a laminated shell theory with embedded piezoelectric layers which was subsequently extended to a finite element formulation (Tzou and Tseng, 1990). Most of the previous approaches have assumed simplified electric and displacement fields through-the-thickness of the laminate, or may have used equivalent force representation of the actuation forces within the laminate.

Limited work has been reported in more generalized models enabling improved representation of local intralaminar and interlaminar effects, including a shear deformation finite element for a three-layer (piezoelectric-structure-piezoelectric) shell (Lammering, 1991), an induced-strain layer-wise displacement field for piezoelectric beams (Robbins and Reddy, 1991) and the generalized plate theory of Liebowitz and Vinson (1991).

This paper describes mechanics for sensory/active composite laminates containing piezoelectric layers. Particular emphasis is placed on the effects of electromechanical coupling at the material level and the introduction of electric potential to the state variables. Hence, the mechanics provide the option for variable displacement and electric potential fields through-the-thickness of the laminate. Because of the unified representation of the electromechanical state, the mechanics have the inherent capability to model the coupled electromechanical response of composite laminates with both sensory and active piezoelectric elements, and may handle general electromechanical boundary conditions. In addition, the mechanics combine the unique features for accurate predictions of intralaminar and interlaminar mechanical strains, electric potential fields, and delamination initiation. As a result, the mechanics are envisioned to be particularly suitable for health monitoring applications.

Two separate theories have been developed and are briefly described herein. In the first approach, the transverse displacement component is assumed to be constant through-the-thickness; in the second, the transverse displacement is variable for the representation of interlaminar normal strains. In both approaches, the in-plane displacements and the electrostatic potential is assumed to have arbitrary piecewise linear variations through the thickness of the laminate. The governing equations and the evolving methods, which result in approximate formulations for the quasi-static and dynamic response are described along these techniques.

Representative validations and applications of the method on composite beams are included. The applications illustrate the capacity of the mechanics to accurately model displacement, electric potential and stress/strain fields through-the-thickness of the laminate, as well as, other global structural characteristics in both active and sensory beams. The results also indicate the ranges of applicability and limitations of simplified mechanical models of sensory/active composites. The capability of the sensors to detect the stress field in the laminate is investigated. Finally, the capability to predict vibrational characteristics under various electrical boundary conditions is illustrated.

METHOD

In this section, the governing equations for a composite beam structure with embedded piezoelectric sensors/actuators are described. The coupled material equations for each ply are first presented in a unified way which may represent either piezoelectric or passive composite layers. The discrete-layer laminate mechanics are subsequently presented. Finally, as a first structural application, a composite beam theory is developed by eliminating one of the in-plane displacement components and modifying the elastic stiffnesses.

Unified Material Representation

The constitutive equations of a monoclinic material, which can represent an orthotropic (transversely isotropic) layer at an off-axis orientation are (Mason, 1950):

$$\begin{aligned}\sigma_i &= Q_{ik}^E S_k - e_{ik} E_k \\ D_i &= e_{ik} S_k + \epsilon_{ik}^S E_k\end{aligned}\tag{1}$$

where: σ_i and S_i are the mechanical stress and strain vectors; E_k is the electric field vector; D_i is the electric displacement component; Q_{ij} is the elastic stiffness tensor (under constant voltage); e_{ij} is the piezoelectric tensor; and ϵ_{ij} is the electric permittivity tensor of the material (under constant strain). The components of the elastic, piezoelectric and dielectric stiffness matrixes for a piezoelectric solid are given by Mason (1950).

Equation (1) completely characterize the electromechanical response of a piezoelectric layer. Clearly, the piezoelectric matrix $[e]$ couples the elastic and electric terms. It is pointed out, that the first equation may be used to represent active piezoelectric elements, however, the second equation is also needed to represent sensory response. Also, equation (1) may decouple to represent either passive off-axis composite plies of elastic stiffness $[Q_c]$ ($[e] = [\epsilon] = 0$), or the capacitance of electric components of ($[\epsilon] \neq 0$, $[Q_c] = [e] = 0$).

The electric field components are related to the electrostatic potential (or voltage) by the equations

$$E_k = -\phi_{,x_k}\tag{2}$$

Additionally, the engineering strain is given by

$$S_{ij} = u_{i,x_j} + u_{j,x_i}\tag{3}$$

The electric enthalpy H_l describing the amount of energy stored in the material is defined as:

$$H_l = \frac{1}{2} Q_{kl} S_k S_l - e_{kl} E_k S_l - \frac{1}{2} \epsilon_{kl} E_k E_l\tag{4}$$

The first variation of electric enthalpy in equation (4) yields

$$\delta H_l = Q_{kl} S_k \delta S_l - e_{kl} E_k \delta S_l - e_{kl} \delta E_k S_l - \epsilon_{kl} E_k \delta E_l\tag{5}$$

Combination of equations (3) and (5) yields the variation of electric enthalpy as function of the material properties and state variables

$$\begin{aligned}
\delta H_I = & C_{11}u_x\delta u_x + C_{12}u_x\delta v_y + C_{13}u_x\delta w_z + C_{16}u_x(\delta u_y + \delta v_x) + \\
& C_{12}v_y\delta u_x + C_{22}v_y\delta v_y + C_{23}v_y\delta w_z + C_{26}v_y(\delta u_y + \delta v_x) + \\
& C_{13}w_z\delta u_x + C_{23}w_z\delta v_y + C_{33}w_z\delta w_z + C_{36}w_z(\delta u_y + \delta v_x) + \\
& C_{44}(v_z + w_y)(\delta v_z + \delta w_y) + C_{45}(v_z + w_y)(\delta u_z + \delta w_x) + \\
& C_{45}(u_z + w_x)(\delta v_z + \delta w_y) + C_{55}(u_z + w_x)(\delta u_z + \delta w_x) + \\
& C_{16}(u_y + v_x)\delta u_x + C_{26}(u_y + v_x)\delta v_y + \\
& C_{36}(u_y + v_x)\delta w_z + C_{66}(u_y + v_x)(\delta u_y + \delta v_x) - \\
& e_{14}\delta E_1(v_z + w_y) - e_{15}\delta E_1(u_z + w_x) - e_{24}\delta E_2(v_z + w_y) - \\
& e_{25}\delta E_2(u_z + w_x) - e_{31}\delta E_3u_x - e_{32}\delta E_3v_y - \\
& e_{33}\delta E_3w_z - e_{36}\delta E_3(u_y + v_x) - \\
& \epsilon_{11}E_1\delta E_1 - \epsilon_{22}E_2\delta E_2 - \epsilon_{33}E_3\delta E_3 - \\
& e_{14}E_1(\delta v_z + \delta w_y) - e_{15}E_1(\delta u_z + \delta w_x) - e_{24}E_2(\delta v_z + \delta w_y) - \\
& e_{25}E_2(\delta u_z + \delta w_x) - e_{31}E_3\delta u_x - e_{32}E_3\delta v_y - \\
& e_{33}E_3\delta w_z - e_{36}E_3(\delta u_y + \delta v_x)
\end{aligned} \tag{6}$$

The inclusion of all forms of stored energy of the piezoelectric layers is obvious in the above equations.

Piezoelectric Composite Laminates

In order to represent the strong heterogeneity in composite laminates with embedded piezoelectric layers, a discrete-layer laminate theory was developed. Discrete-layer approaches are suitable for capturing effects induced from the discontinuous variation of properties and anisotropy through-the-thickness of the laminate (Saravanan and Pereira, 1992) and can provide superior calculations of intralaminar and interlaminar strains (Reddy, 1987). Consequently, the suitability of discrete-layer theories for smart laminates, which entail additional heterogeneity from piezoelectric layers and induced strain actuation, cannot be understated. An additional advantage of the proposed discrete-layer piezoelectric theory is the inherent option to control the representation of electric potential and displacement fields (fig. 1). At the lower limit, the method may be reduced to either Kirchhoff-Love assumptions (CLPT) or first order (Mindlin) shear theory.

The laminate mechanics described in this section were simplified to obtain solutions for composite beams with embedded piezoelectric layers. Hence, only axial variation of the state variables is assumed herein,

$$u = u(x,z), \quad v = 0, \quad w = w(x,z), \quad \phi = \phi(x,z) \tag{7}$$

In this context, piecewise continuous approximations (fig. 1) of the electromechanical state (displacements and electric potential) through-the-thickness of the laminate are proposed:

$$\begin{aligned}
u(x,z,t) &= \sum_{j=1}^N U_j(x,t) \tilde{\Psi}_j^u(z) \\
w(x,z,t) &= \sum_{j=1}^N W_j(x,t) \tilde{\Psi}_j^w(z) \\
\phi(x,z,t) &= \sum_{j=1}^N \Phi_j(x,t) \tilde{\Psi}_j^\phi(z)
\end{aligned} \tag{8}$$

The N interpolation functions $\Psi_j(z)$ represent piecewise linear through-the-thickness approximations. By setting Ψ^w equal to unity, a constant transverse displacement is assumed, otherwise, the transverse displacement varies through the thickness.

Two unique features of the method are stressed thereafter. Clearly, the formulation entails the flexibility to model smart composite laminates with either interlaminarly embedded or "add on" piezoelectric layers. Additionally, the detail of the through-the-thickness approximations may be controlled by changing the number of functions $\Psi_j(z)$, hence variable degrees of accuracy are attained depending on the specific application. For $N = 2$ and $\Psi^w = 1$, the method reduces to either the first-order shear (Mindlin) laminate theory or the classical (Kirchoff-Love) laminate plate theory ($\{u_1\} = -\{u_2\}$). The electric enthalpy of the laminate is:

$$H_L = \int_{-h}^h H_1 dz \tag{9}$$

where h is the half-thickness of the cross section. Combination of equations (6), (8), and (9) and integration through-the-thickness results in the following generalized laminate matrixes describing the stiffness of the laminate (under constant electric field),

$$\begin{aligned}
A_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} C_{ij} \Psi^m(z) \Psi^n(z) dz & i,j &= 1,2,6 \\
A_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} C_{ij} \Psi^m(z) \Psi_{,z}^n(z) dz & i &= 1,2,6, \quad j = 3 \\
A_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} C_{ij} \Psi_{,z}^m(z) \Psi^n(z) dz & i,j &= 4,5 \\
B_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} C_{ij} \Psi_{,z}^m(z) \Psi^n(z) dz & i,j &= 4,5 \\
D_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} C_{ij} \Psi^m(z) \Psi^n(z) dz & i,j &= 4,5
\end{aligned} \tag{10}$$

the piezoelectric properties,

$$\begin{aligned}
 E_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} e_{ij} \Psi^m(z) \Psi^n(z) dz \quad i = 1, 2 \quad j = 4, 5 \\
 &\quad \text{and } i = 3 \quad j = 1, 2, 6 \\
 \tilde{E}_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} e_{ij} \Psi^m(z) \Psi^n(z) dz \quad i = 1, 2 \quad j = 4, 5 \\
 \hat{E}_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} e_{ij} \Psi_z^m(z) \Psi_z^n(z) dz \quad i, j = 3
 \end{aligned} \tag{11}$$

and the dielectric properties (under constant strain) of the laminate,

$$\begin{aligned}
 G_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} \epsilon_{ij} \Psi^m(z) \Psi^n(z) dz \quad i = j = 1, 2 \\
 G_{ij}^{mn} &= \sum_{l=1}^L \int_{z_l}^{z_{l+1}} \epsilon_{ij} \Psi_z^m(z) \Psi_z^n(z) dz \quad i = j = 3
 \end{aligned} \tag{12}$$

Sensory/Active Composite Beams

The laminate mechanics are further extended to obtain structural solutions for beams. By assuming an additional in-plane approximations using the one-dimensional function $\psi_j(x)$, equation (8) take the following form:

$$\begin{aligned}
 u(x, z, t) &= \sum_{i=1}^m \sum_{j=1}^n U_{ji}(t) \Psi_i^u(x) \tilde{\Psi}_j^u(z) \\
 w(x, z, t) &= \sum_{i=1}^m \sum_{j=1}^n W_{ji}(t) \Psi_i^w(x) \tilde{\Psi}_j^w(z) \\
 \phi(x, z, t) &= \sum_{i=1}^m \sum_{j=1}^n \Phi_{ji}(t) \Psi_i^\phi(x) \tilde{\Psi}_j^\phi(z)
 \end{aligned} \tag{13}$$

The assumed formulation allows both global (Fourier series, Legendre polynomials, etc.) and local (finite element) solutions to be implemented along the x axis. A local approximation method is presented herein.

The variational formulation of the equations of motion for the sensory/active composite structure, is provided by the Hamilton's principle (Tiersten, 1969).

$$\delta \int_{t_0}^t dt \int_V \left[\frac{1}{2} \rho \dot{u}_j \dot{u}_j - H_l(S_k, E_k) \right] dV + \int_{t_0}^t dt \int_S (\bar{t}_k \delta \bar{u}_k - q \delta \phi) dS = 0 \tag{14}$$

where t is time, V and S are the volume and the bounding surface respectively of the solid, ρ is the density of the material, u_i represents the displacement components, t bar and q are surface tractions and surface charge respectively, δ is the variational operator, the $\dot{}$ superscript represents differentiation with respect to time, and ϕ is the electrostatic potential.

Integrating the variational formulation (eq. (14)) through-the-thickness, then combining equations (10) to (14), the following discrete coupled system results, which describes the electrodynamic response of the smart structure:

$$\begin{bmatrix} [M_{uu}] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\ddot{U}\} \\ \{\ddot{\phi}\} \end{Bmatrix} + \begin{bmatrix} [K_{uu}] & [K_{ue}] \\ [K_{eu}] & [K_{ee}] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{\phi\} \end{Bmatrix} = \begin{Bmatrix} \{F(t)\} \\ \{Q(t)\} \end{Bmatrix} \quad (15)$$

where $\{U\} = \{\{u\}, \{w\}\}$ is the structural displacement vector containing both in-plane and lateral displacements; $\{\phi\} = \{\{\phi^S\}; \{\phi^A\}\}$ is the electric state vector describing the voltage output at the sensors (superscript S) and the applied voltage at the actuators (superscript A), $F(t)$ is external mechanical load vector, and $Q(t)$ is the electric charge at the terminals. The submatrixes K_{uu} , K_{ue} , K_{ee} in equation (15) represent the structural, piezoelectric, and dielectric components of the structure, the submatrix M_{uu} represents the structural mass of the structure. The elements of these matrixes are additional submatrixes whose elements are determined by multiplying the laminate matrixes (eqs. (10) to (12)) by the various shape functions or their derivatives and then integrating along the axis, as determined by the variational statement. Local linear interpolation functions along the x -axis are used in this paper.

The electric potential vector in equation (15) is further partitioned in terms of sensory and active components,

$$\begin{bmatrix} [M_{uu}] & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \{\ddot{U}\} \\ \{\ddot{\phi}^S\} \end{Bmatrix} + \begin{bmatrix} [K_{uu}] & [K_{ue}^{SS}] \\ [K_{eu}^{SS}] & [K_{ee}^{SS}] \end{bmatrix} \begin{Bmatrix} \{U\} \\ \{\phi^S\} \end{Bmatrix} = \begin{Bmatrix} \{F(t)\} - [K_{ue}^{SA}]\{\phi^A\} \\ \{Q^S(t)\} - [K_{ee}^{SA}]\{\phi^A\} \end{Bmatrix} \quad (16)$$

where the left-hand term includes the unknown response of the structure in terms of displacements and electric potential at the sensors, while the right-hand includes the excitation of the structure in terms of mechanical loads and applied voltages on the actuators. The electric charge at the sensors Q^S remains constant (practically open-circuit conditions) and is assumed equal to a zero initial value.

The resultant coupled form of the governing equations and the inclusion of electric potential in the state variables is emphasized once more. Besides the gains in accuracy from the electromechanical coupling in equation (16), the obvious advantages is the capability of the mechanics to model the response of the smart structure either: in "active" mode, that is with specified voltages $\Delta\phi^A$ applied across the piezoelectric layers to induce a desirable deflection/strain state; or in "sensory" mode where displacements or mechanical loads are applied to the structure and the resultant voltage or charge is monitored; or in combined "active/sensory" mode. This capability is further illustrated in the following applications.

Finally, the dynamic system of equation (16) is condensed in the following form, which provides the structural displacements independently,

$$[M_{uu}]\{\ddot{U}\} + ([K_{uu}] - [K_{ue}^{SS}][K_{ee}^{SS}]^{-1}[K_{eu}^{SS}])\{U\} = \{F(t)\} + ([K_{ue}^{SS}][K_{ee}^{SS}]^{-1}[K_{ee}^{SA}] - [K_{ue}^{SA}])\{\phi^A\} \quad (17)$$

and the electric potentials at the sensors:

$$\{\phi^s\} = -[K_{ee}^{ss}]^{-1}([K_{eu}^{ss}]\{U\} - [K_{ee}^{sa}]\{\phi^a\}) \quad (18)$$

Note, that the active voltage term in the right-hand side of equation (17) resembles the equivalent induced force formulation. However, both the equivalent stiffness matrix (left-hand side) and the equivalent induced-force at the active layers in the right-hand side of equation (17) include additional terms, which usually are not included in the "induced-strain" formulations to date.

APPLICATIONS AND DISCUSSION

This section presents results from several representative problems. The accuracy and validity of the developed mechanics is illustrated by comparisons with the generalized model of Robbins and Reddy (1991). All applications were focused on a three-layer active/sensory cantilever beam configuration (fig. 2) with either an aluminum or a unidirectional Gr/Epoxy (T300/934) composite layer. The beam is 6 in. long and is composed of a 0.6-in. thick passive layer bonded to a 0.06-in. thick piezoelectric layer using a 0.01-in. thick adhesive layer. This is a simple geometry that provides valuable insight into the problem, as the piezoelectric layer can be configured to act either as sensor or as actuator. More complicated configurations will be studied in the near future. Unless otherwise stated the piezoelectric material is assumed to be the commercially available piezoceramic PZT-4 (Vernitron Corp.) with properties provided by the manufacturer's data sheet (table I).

In all static cases, a total of 29 layers were used to model the thickness of the beam, with 16/5/8 layers used for the aluminum (or composite)/adhesive/piezoelectric, respectively. In the calculation of the natural frequencies 10 layers were used through-the-thickness with 6/2/2 layers used for the aluminum/adhesive/ piezoelectric, respectively. The elements were of equal thickness within each layer. The length of the beam was modeled using 25 linear elements.

Quasi-Static Response

Verification.—Predicted displacement profiles for a three-layer "active" cantilever beam are presented and compared with results reported by Robbins and Reddy (1991) with material properties shown in table I. The reported results provide one of the best available comparison problems for the theories developed in this work. Moreover, they have reported comparisons with other simplified mechanics. The shortcomings of simplified mechanics are also obvious in the obtained results.

In their work, Robbins and Reddy provide only mechanical properties of the piezoelectric layer, because they assumed 0.1-percent induced-strain actuation. However, the current method requires a more complete set of properties, hence, the piezoelectric and dielectric properties of PZT-4 were used for the piezoelectric layer (table I). To impose equivalent boundary conditions for the electric potential it was calculated that 12.4 kV applied across the free surfaces of the piezoelectric layer, induced 0.1-percent average axial strain under stress-free conditions. Consequently, a constant electric potential of 12.4 kV was then applied on the upper surface of the PZT-4 layer, while the lower surface was assumed grounded (0 V).

The predicted values of the transverse and axial (mid-point) displacement components at the free-end are shown in table II. There is fairly good agreement between the theories with constant and variable lateral displacement fields. The present theories yield slightly lower displacement values, as they provide more complete representation of all energy forms (strain, piezoelectric, and electric energy), hence, only part of the electric

enthalpy is converted to elastic strain energy while a small portion is stored in the electric field. Contrary to this, the induced strain approach assumes that all electric enthalpy is converted to elastic strain energy, hence results in larger deflections.

Active beams.—The present cases investigate further the response of the three-layer beam (fig. 2) when an electric potential differential of 12.4 kV is applied between the upper and lower surfaces of the piezoelectric layer. The assumed material properties are shown in table I. Passive layers of either aluminum or unidirectional T300/934 composite were considered, although due to space limitations, mostly results for the composite beam are presented.

The axial variations of the transverse displacement and applied voltage of the T300/934 active composite beam are shown in figure 3. The predicted through-the-thickness fields of the axial displacement u , electric potential, axial normal strain (S_x) and shear strain (S_{xy}) are shown in figures 4 and 5. The axial variations of the axial strain at the bottom and top of the passive layer, the center of the adhesive and the top of the piezoelectric layer are shown in figure 6. The interlaminar shear strain in the adhesive layer is also shown in figure 6.

For cross sections away from the free-edge the displacement and strain fields are linear and the Kirchhoff-Love assumptions appear reasonable, however, significant deviations appear near the free-edge and the strains predicted by the present theory are dramatically different. The differences were higher for the case of the T300/934 composite than the aluminum beam, suggesting the suitability of the present method for smart structures with composite passive plies.

Moreover, the predicted results in figures 5 and 6 indicate the critical modes in an active composite structure. The high interlaminar shear strains in the adhesive layer near the free-end suggest possible initiation of a delamination crack between the active layer and the composite substructure. This is expected to be a primary failure mode, and cannot be predicted with simplified laminate theories. The axial mechanical stresses in the all three materials are typically higher away from the free-end may result in possible tensile failure there depending on the applied external loads. The previous observations illustrate further the significance of the present theory for the analysis and design of reliable active structures.

Sensory beams.—The response of the three-layer cantilever beam operating in a sensory mode with a transverse load (1000 lb) applied at the free-end is investigated here. The lower surface of the piezoelectric layer is grounded (0 V) and the signal is acquired from electrodes on the top surface. The applications demonstrate the inherent capacity of the theory to model sensory structures and the interactions between passive and piezoelectric plies. Of particular interest is the capability to predict the resultant strain/stress fields in the beam and the possibility to correlate the strains to the resultant voltage distribution. Such capability is essential for "health monitoring" applications where continuous monitoring of strain fields may enhance structural reliability.

The resultant transverse displacements and electric potential (top surface) for both aluminum and T300/934 beams are shown in figure 7. The electric potential and axial displacement fields across various cross sections of the sensory T300/934 beam are shown in figure 8. The electric potential is low near the tip and increases almost linearly towards the clamped edge. The resultant strain fields at various cross sections of the composite sensory beam are shown in figures 9 and 10. The axial strain varies almost linearly along the axis of the beam while the interlaminar strains are high and constant. It is mentioned that the deviations from the assumptions of classical plate theory near the fixed-end were more predominant in the composite than the aluminum beam.

It seems that sufficient correlation exists between the electric potential and the normal strains in the beam and the present mechanics may not only quantify these important relationships, but provide the link for relating sensory voltage to strains. Similar correlations may be established with the interlaminar shear in sections away from the clamped end. Hence, the sensory voltage may be used in connection with the mechanics to monitor the

critical strain/stresses both axially and through-the-thickness of the beam, and provide estimations about the location and type of eminent damage.

Free-Vibration Response

The following applications illustrate the additional capabilities of the mechanics to predict the free-vibration response of smart composite structures under various electromechanical boundary conditions. The natural frequencies of the cantilever aluminum beam with a PZT-4 piezoceramic layer (fig. 2) are calculated using equation (18). Two cases of electric boundary conditions were considered: (1) open-circuit conditions where the voltage on the top surface of the piezoceramic was free; and (2) closed-circuit conditions where the electric potential on the top surface was also constrained to be zero. The predicted first eight natural frequencies are shown in table III. The differences in natural frequency predictions between the open- and closed-circuit conditions are obvious. The differences between variable $w(z)$ and constant $w(z)$ may be attributed to Poisson's effects influencing both stiffness and electric capacitance of the piezoelectric layer. Reported results by Robbins and Reddy (1991) are also shown in the same table, however, these predictions include only the elastic terms. The present mechanics yield higher natural frequencies, which is consistent with the static results (table II). Additional results in this area will be presented in future work.

SUMMARY

Discrete-layer mechanics were developed and described for simulating the coupled electromechanical response of composite structures with embedded piezoelectric sensory and active layers. The capability and versatility of the mechanics to model the coupled quasi-static and free dynamic response of composite structures operating either in an active (applied voltages) or sensory (applied force/displacement) mode was demonstrated on aluminum and T300 Gr/Epoxy cantilever beams with a piezoceramic layer.

The mechanics yielded excellent predictions of the global static response and natural frequencies of the beam under various electrical conditions. Also, the mechanics yielded excellent local representations of intralaminar and interlaminar strain fields which depicts their suitability for monitoring the safety margins against possible failure. Significant differences with other simplified laminate theories were predicted, especially in the calculation of strains in active beams. The results indicate that by using the developed theories, strong correlations may be established between the electric voltage at the sensors and the displacements/strains within the passive structural components, the active layers, and their interphase. Overall, it was shown that the developed mechanics may provide unique advantages in simulating the response of adaptive structures. Additional unique advantages were demonstrated towards the development of smart composites with improved "damage resilience." Future work is directed towards these areas.

ACKNOWLEDGMENT

Part of this work was performed during an ASEE Summer Faculty Fellowship of the second author at the NASA Lewis Research Center. The work was monitored by Mr. Dale A. Hopkins.

REFERENCES

- Bailey, T.; and Hubbard, J.E.: Distributed Piezoelectric Polymer Active Vibration Control of Cantilever Beam. AIAA J. Guid. Control, Vol. 8, 1985, pp. 606–610.
- Crawley, E.F.; and de Luis, J.: Use of Piezoelectric Actuators as Elements of Intelligent Structures. AIAA J., Vol. 25, 1987, pp. 1373–1385.
- Dimitriadis, E.K.; Fuller, C.R.; and Rogers, C.A.: Piezoelectric Actuators for Distributed Noise and Vibration Excitation of Thin Plates. Proceedings of ASME Failure Prevention and Reliability Conference, Montreal, 1989, pp. 223–233.
- Fanson, J.L.; and Chen, J.C.: Structural Control by the Use of Piezoelectric Active Members. Proceedings of NASA/DOD Control-Structures Interaction Conference, NASA CP-2447, Part II, 1986.
- Lammering, R.: The Application of a Finite Shell Element for Composites Containing Piezoelectric Polymers in Vibration Control. Computers and Structures, Vol. 41, 1991, pp. 1101–1109.
- Lazarus, K.B.; and Crawley, E.F.: Induced Strain Actuation of Composite Plates. GTL Report No. 197, Massachusetts Institute of Technology, Cambridge, MA, 1989.
- Lee, C.-K.: Theory of Laminated Piezoelectric Plates for the Design of Distributed Sensors/Actuators. Part I: Governing Equations and Reciprocal Relationships. J. Acous. Soc. Am., Vol. 87, 1990, pp. 1144–1158.
- Lee, C.-K.; and Moon, F.C.: Laminated Piezopolymer Plates for Torsional Bending Sensors and Actuators. J. Acous. Soc. Am., Vol. 85, 1989, pp. 2432–2439.
- Lee, C.-K.; and Moon, F.C.: Modal Sensors/Actuators. ASME J. Appl. Mech., Vol. 57, 1990, pp. 434–441.
- Liebowitz, M.; and Vinson, J.R.: Intelligent Composites: Design and Analysis of Composite Material Structures Involving Piezoelectric Material Layers. Part A-Basic Formulation. Technical Report CCM 91–54, University of Delaware Center for Composite Materials, 1991.
- Mason, W.P.: Piezoelectric Crystals and Their Applications to Ultrasonics. D. Van Nostrand Co., New York, 1950.
- Reddy, J.N.: A Generalization of Displacement-Based Laminate Theories. Communications in Applied Numerical Methods, Vol. 3, 1987, pp. 173–181.
- Robbins, D.H.; and Reddy, J.N.: Analysis of Piezoelectrically Actuated Beams Using a Layer-Wise Displacement Theory. Computers and Structures, Vol. 41, 1991, pp. 265–279.
- Saravanos, D.A.; and Pereira J.M.: Effects of Interply Damping Layers on the Dynamic response of Composite Plates. AIAA J. Vol. 30, No. 12, Dec. 1992, pp. 2906–2913.
- Tiersten, H.F.: Linear Piezoelectric Plate Vibrations. Plenum Press, New York, 1969.
- Tzou, H.S.: Distributed Sensing and Controls of Flexible Plates and Shells Using Distributed Piezoelectric Elements. J. Wave-Mater. Inter., Vol. 4, 1989, pp. 11–29.

- Tzou, H.S.; and Gadre, M.: Theoretical Analysis of a Multi-Layered Thin Shell Coupled with Piezoelectric Shell Actuators for Distributed Vibration Controls. *J. Sound and Vibr.*, Vol. 132, 1989, pp. 433–450.
- Tzou, H.S.; and Tseng, C.I.: Distributed Piezoelectric Sensor/Actuator Design for Dynamic Measurement/Control of Distributed Parametric Systems: A Piezoelectric Finite Element Approach. *J. Sound and Vibr.*, Vol. 138, 1990, pp. 17–34.
- Venneri, S.L.; and Noor, A.K.: Overview and Major Characteristics of Future Aeronautical and Space Systems. AIAA Paper 92–2441–CP, Proceedings, 33rd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Dallas, TX, 1992, pp. 1924–1934.
- Wang, B.T.; Dimitriadis, E.K.; and Fuller, C.R.: Active Control of Panel Related Noise Using Multiple Piezoelectric Actuators. *J. Acous. Soc. Am.*, Vol. 86 (S1), 1989, p. S84.
- Wang, B.-T.; and Rogers, C.A.: Laminate Plate Theory for Spatially Distributed Induced Strain Actuators. *J. Compos. Mater.*, Vol. 25, 1991, pp. 433–452.

TABLE I.—GEOMETRY AND MATERIAL DATA FOR BEAM

Property	Aluminum	T300/934	Adhesive	Piezoelectric (Robbins, 1991)	PZT-4 (Vernitron)
E_{11}^E , Mpsi	10	18.3	1	10	11.9
E_{33}^E , Mpsi	10	1.14	1	7	11.9
ν_{13}	0.25	0.275	0.4	0.25	0.31
ν_{23}	0.25	0.466	0.4	0.175	0.31
G_{13}^E , Mpsi	4	0.5	0.357	3	4.5
d_{31} , in./V	0	0	0	-----	$-4.8 \cdot 10^{-9}$
d_{33} , in./V	0	0	0	-----	$11.2 \cdot 10^{-9}$
ϵ_{31}^S , psi/V ²	0	0	0	-----	$2.14 \cdot 10^{-9}$
Thickness, in.	0.6	0.6	0.01	0.06	0.06
Length, in.	0.6	6.0	6.0	6.0	6.0

TABLE II.—PREDICTED TIP DISPLACEMENTS FOR ACTIVE BEAM
(AT MID-PLANE)

Component, normalized	Present ($w = \text{cons}$), 12.4 kV	Robbins, 0.1% strain	Present ($w = w(z)$), 12.4 kV	Robbins, 0.1% strain
$u/2h$ (10^{-4})	7.9986	8.0239	7.895	7.8699
$w/2h$ (10^{-4})	1.9388	2.0393	1.895	2.0358

TABLE III.—PREDICTED NATURAL FREQUENCIES

[Aluminum with single PZT-4 layer.]

Mode	Present ($w = \text{const}$)		Robbins	Present ($w = w(z)$)		Robbins
	Open	Closed		Open	Closed	
1	544	551	538	545	555	538
2	3238	3275	3193	3232	3286	3181
3	7540	7577	7576	7530	7586	7566
4	8464	8550	8340	8466	8589	8201
5	15275	15416	15017	15309	15501	14434
6	22490	22628	22462	22476	22674	21371
7	23349	23520	23023	23414	23637	22709
8	31945	32192	31220	32096	32395	28886

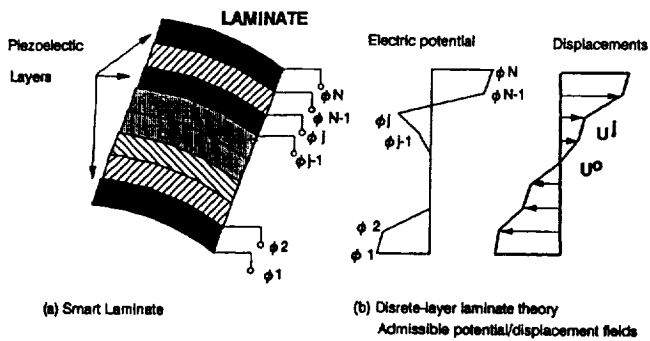


Figure 1.—Typical sensory/active smart laminate. (a) Concept. (b) Kinematic assumptions.

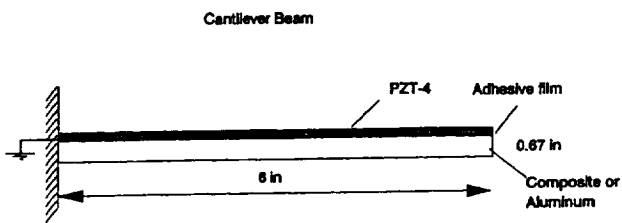


Figure 2.—Cantilever active/sensory composite beam.

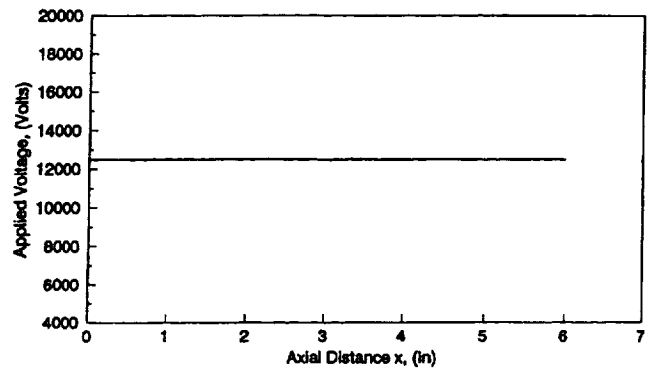
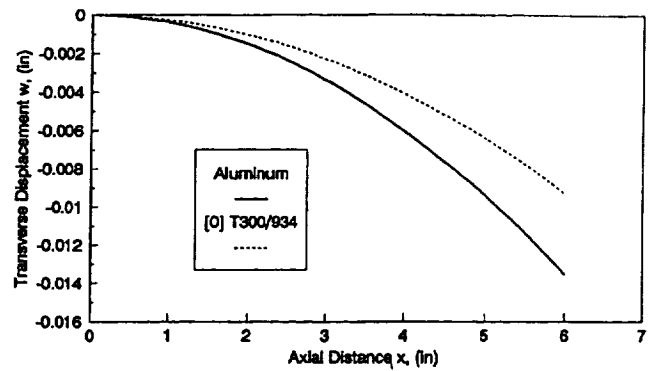


Figure 3.—Predicted transverse displacements in active beams.

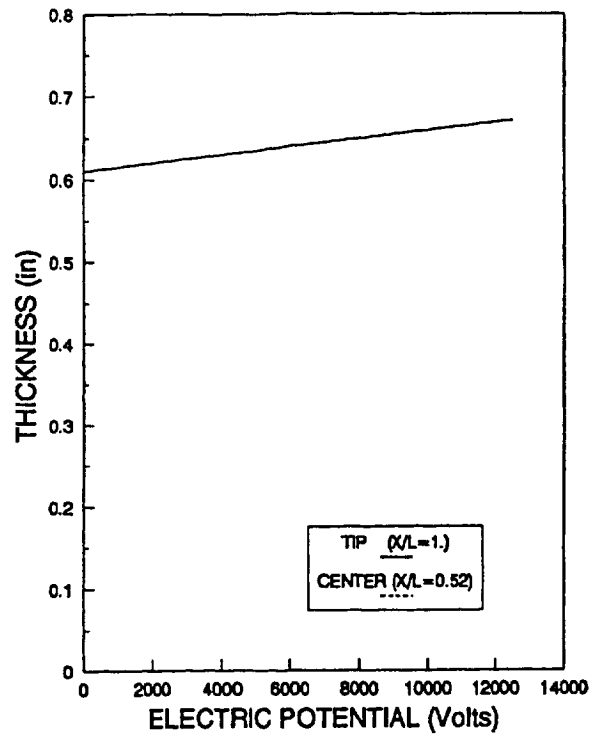
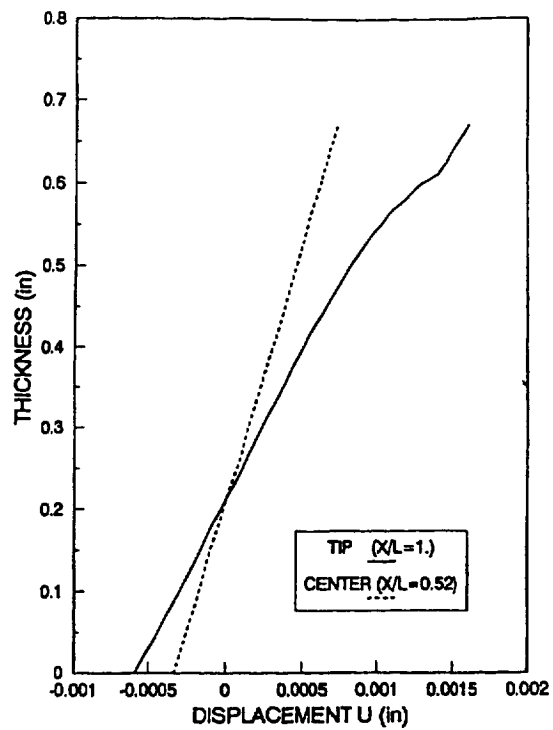


Figure 4.—Axial displacement and electric potential fields in active T300/934 composite beam.

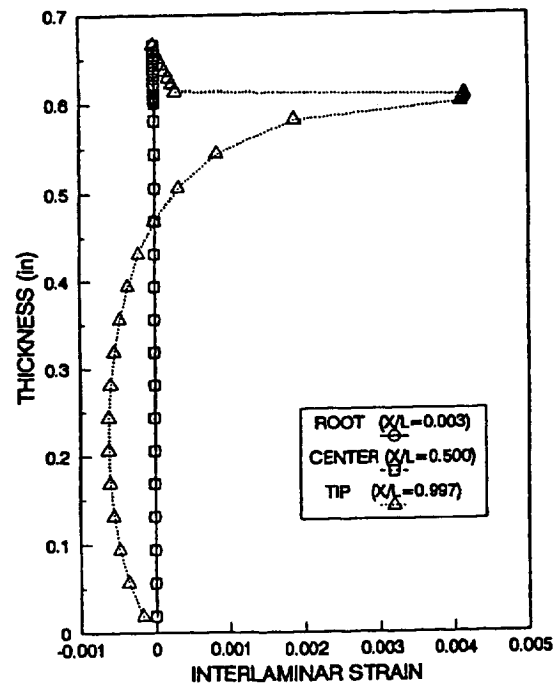
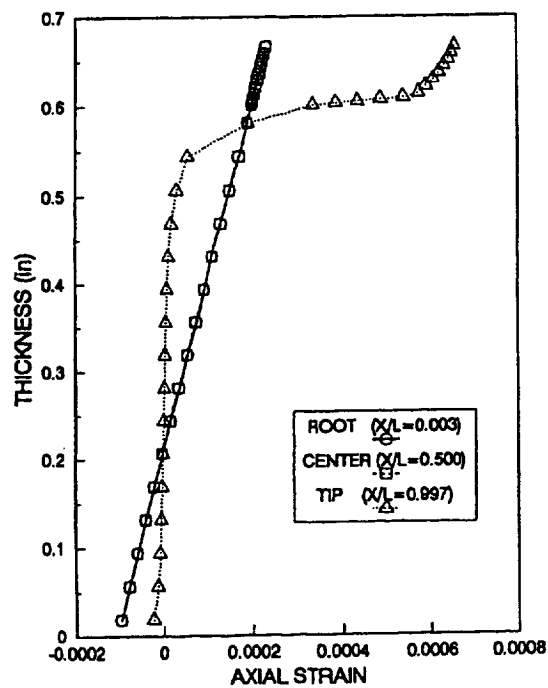


Figure 5.—Through-the-thickness variations of axial and interlaminar shear strains in active T300/934 composite beam.

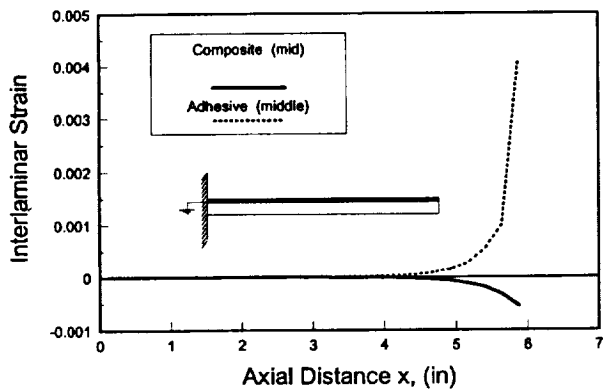
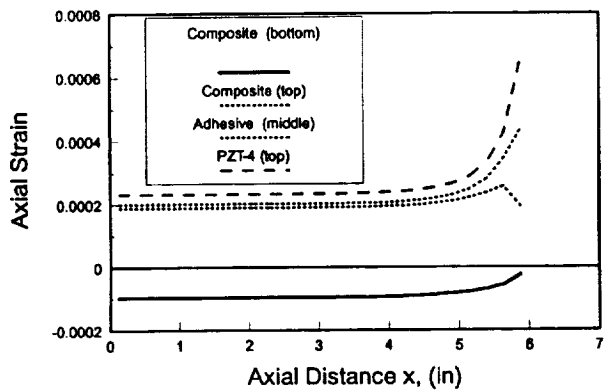


Figure 6.—Axial and interlaminar strains in active T300/934 composite beam.

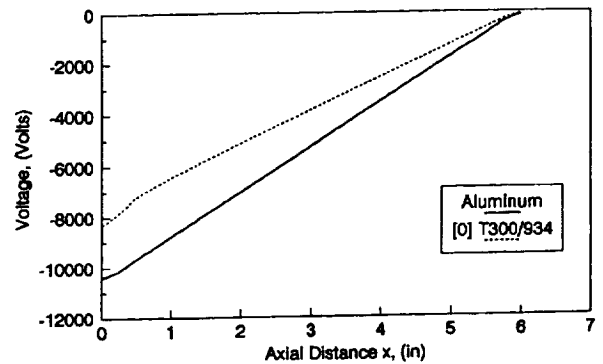
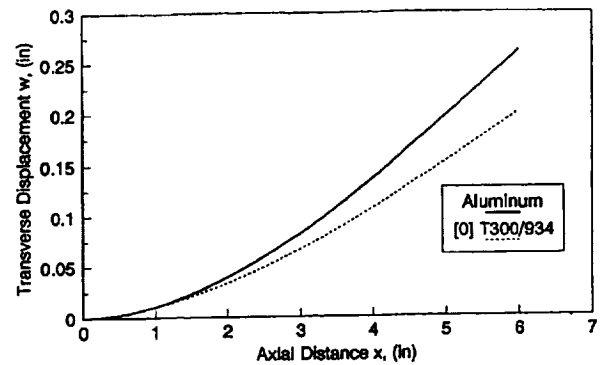


Figure 7.—Resultant transverse displacements and electric potential in top surface of sensory beams. 1000 lbs of transverse force at the free end.

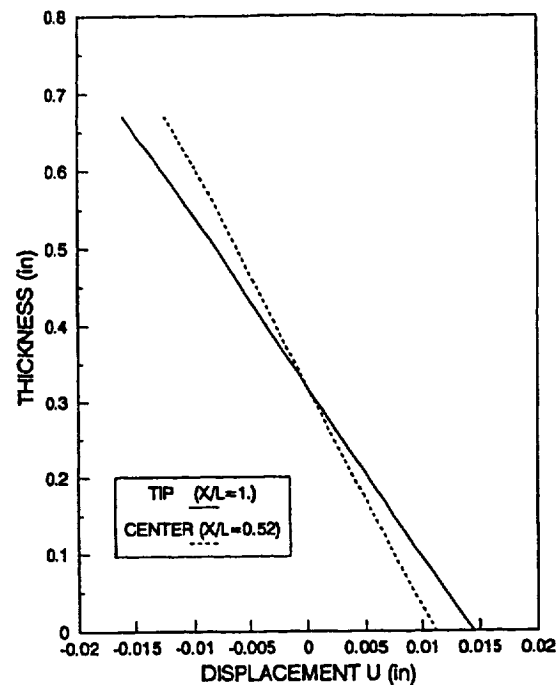
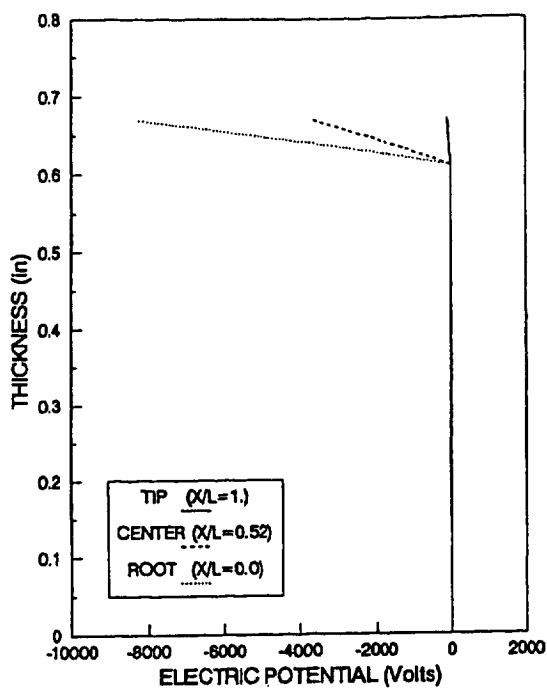


Figure 8.—Axial Displacement and electric potential fields in sensory T300/934 composite beam.

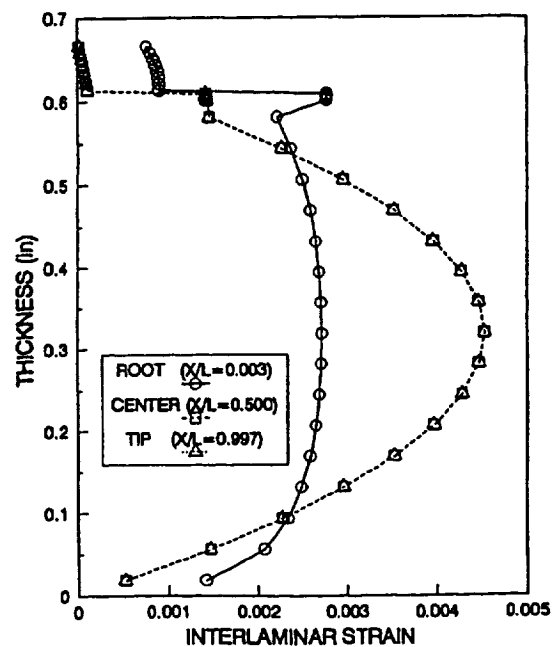
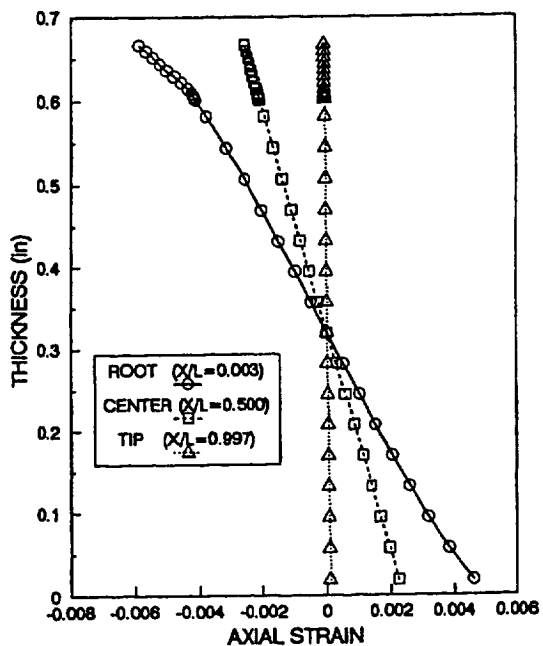


Figure 9.—Through-the-thickness variations of axial and interlaminar shear strains in sensory T300/934 composite beam.

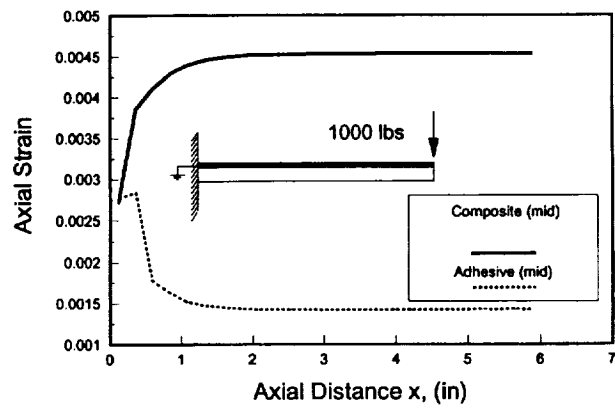
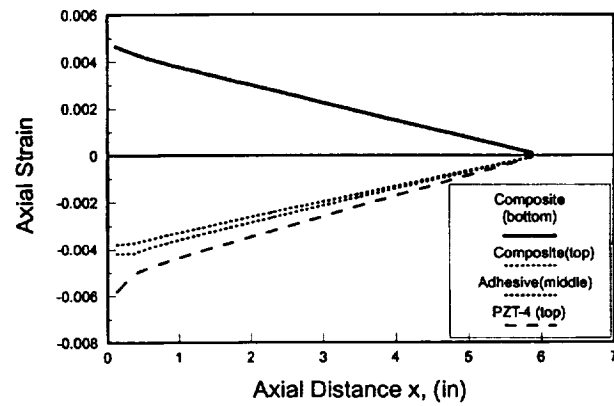


Figure 10.—Axial and interlaminar strains in sensory T300/934 composite beam.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE June 1994		3. REPORT TYPE AND DATES COVERED Final Contractor Report
4. TITLE AND SUBTITLE Coupled Electromechanical Response of Composite Beams With Embedded Piezoelectric Sensors and Actuators			5. FUNDING NUMBERS WU-505-63-5B NCC3-208	
6. AUTHOR(S) D.A. Saravanos and P.R. Heyliger				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Ohio Aerospace Institute 22800 Cedar Point Road Brook Park, Ohio 44142			8. PERFORMING ORGANIZATION REPORT NUMBER E-8728	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135-3191			10. SPONSORING/MONITORING AGENCY REPORT NUMBER NASA CR-195313	
11. SUPPLEMENTARY NOTES Prepared for the ASME Winter Annual Meeting sponsored by the American Society of Mechanical Engineers, New Orleans, Louisiana, November 28--December 3, 1993. D.A. Saravanos, Ohio Aerospace Institute, 22800 Cedar Point Road, Brook Park, Ohio 44142 and P.R. Heyliger, Colorado State University, Civil Engineering Department, Fort Collins, Colorado 80523 and Summer Faculty Fellow at NASA Lewis Research Center (work funded by NASA Cooperative Agreement NCC3-208). Project Manager, Dale A. Hopkins, Structures Division, organization code 5210, (216) 433-3260.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified - Unlimited Subject Category 24			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) Unified mechanics are developed with the capability to model both sensory and active composite laminates with embedded piezoelectric layers. A discrete-layer formulation enables analysis of both global and local electro-mechanical response. The mechanics include the contributions from elastic, piezoelectric, and dielectric components. The incorporation of electric potential into the state variables permits representation of general electromechanical boundary conditions. Approximate finite element solutions for the static and free-vibration analysis of beams are presented. Applications on composite beams demonstrate the capability to represent either sensory or active structures, and to model the complicated stress-strain fields, the interactions between passive/active layers, interfacial phenomena between sensors and composite plies, and critical damage modes in the material. The capability to predict the dynamic characteristics under various electrical boundary conditions is also demonstrated.				
14. SUBJECT TERMS Composites; Laminates; Piezoelectrics; Adaptive structures; Smart materials; Structures; Beams; Sensors; Actuators			15. NUMBER OF PAGES 20	
			16. PRICE CODE A03	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	